Reachability in Two-Dimensional Vector Addition Systems with States is PSPACE-complete

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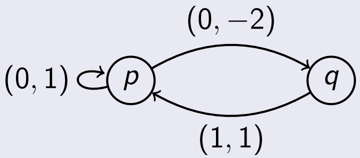
Vector addition system with states (VASS)

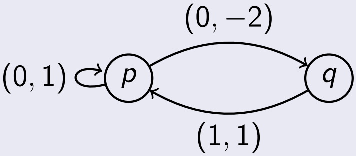
d-VASS:



Vector addition system with states (VASS)

d-VASS:

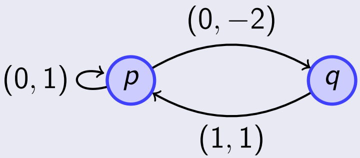
- $d \geq 1$ (***dimension***)



Vector addition system with states (VASS)

d-VASS:

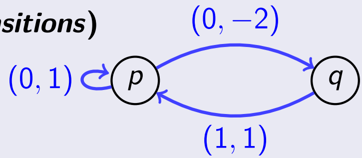
- $d \geq 1$ (*dimension*)
- **Q finite set (*states*)**



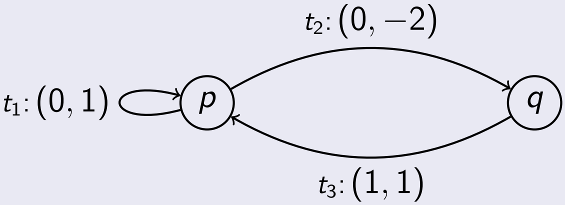
Vector addition system with states (VASS)

d-VASS:

- $d \geq 1$ (*dimension*)
- Q finite set (*states*)
- $\boldsymbol{T \subseteq Q \times \mathbb{Z}^d \times Q}$ **finite (*transitions*)**



Runs



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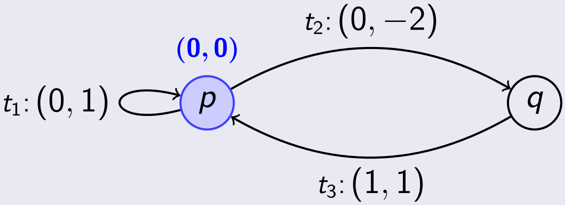
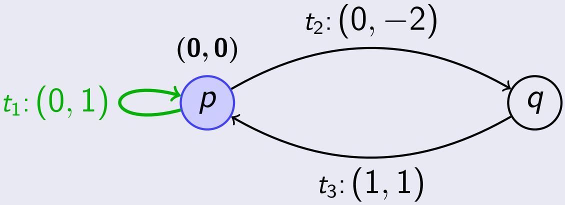


$p(0, 0)$

Runs



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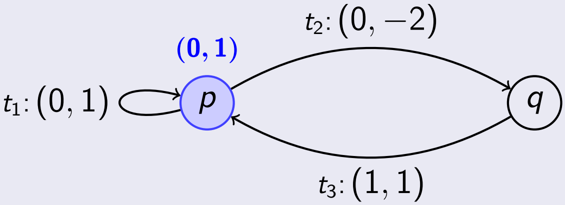
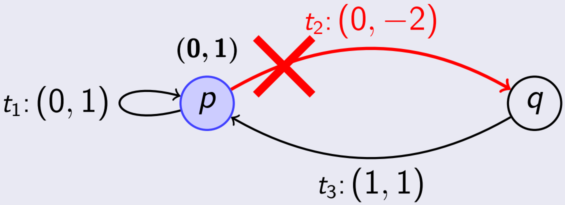
Runs



$p(0,0) \xrightarrow{t_1} p(0,1)$

Runs



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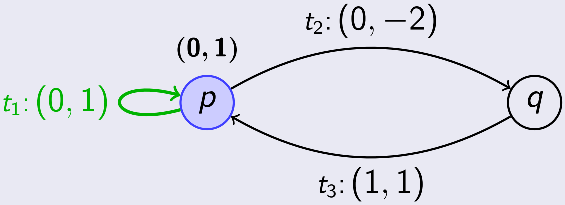
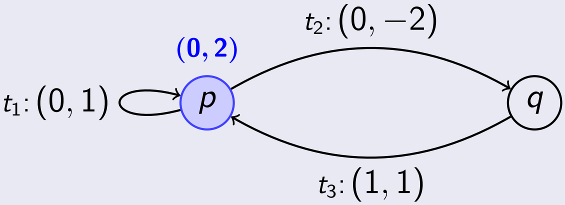


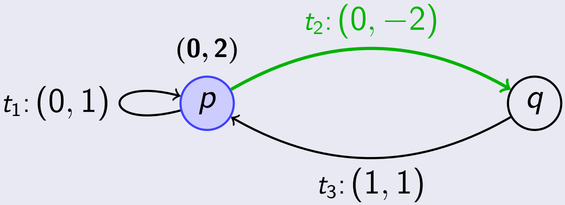
$p(0,0) \xrightarrow{t_1} p(0,1)$

Runs



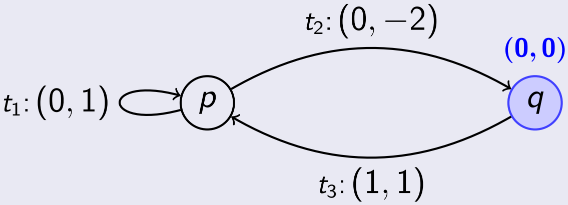
$$p(0,0) \xrightarrow{t_1} p(0,1) \xrightarrow{t_1} p(0,2)$$

Runs



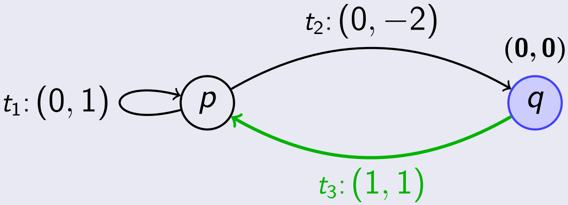
$$p(0,0) \xrightarrow{t_1} p(0,1) \xrightarrow{t_1} p(0,2)$$

Runs



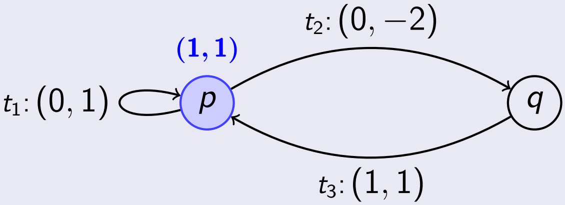
$$p(0,0) \xrightarrow{t_1} p(0,1) \xrightarrow{t_1} p(0,2) \xrightarrow{t_2} q(0,0)$$

Runs



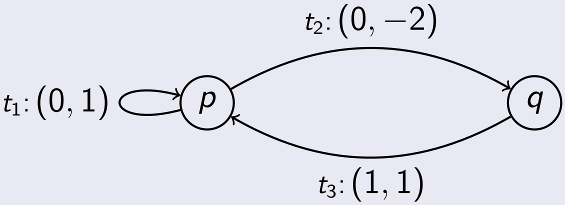
$$p(0,0) \xrightarrow{t_1} p(0,1) \xrightarrow{t_1} p(0,2) \xrightarrow{t_2} q(0,0)$$

Runs



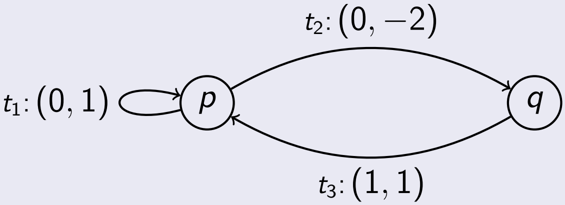
$$p(0,0) \xrightarrow{t_1} p(0,1) \xrightarrow{t_1} p(0,2) \xrightarrow{t_2} q(0,0) \xrightarrow{t_3} p(1,1)$$

Runs



$$p(0,0) \xrightarrow{t_1\,t_1\,t_2\,t_3} p(1,1)$$

Runs



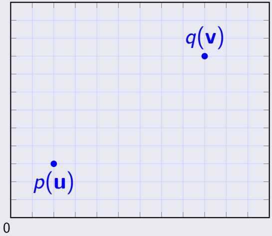
$$p(0,0) \xrightarrow{*} p(1,1)$$

Reachability problem

Input: d-VASS V

Reachability problem

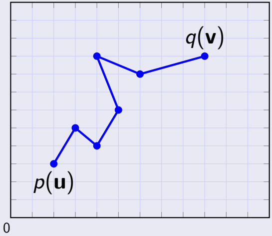
Input: d-VASS V and $p(\mathbf{u}), q(\mathbf{v}) \in Q \times \mathbb{N}^d$



Reachability problem

Input: d-VASS V and $p(\mathbf{u}), q(\mathbf{v}) \in Q \times \mathbb{N}^d$

Question: $p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v})$?



Known results timeline

VASS **2-VASS**

Known results timeline

VASS		2-VASS
	2015	
	2013	
	2012	
	2011	
	2009	
	2004	
	1992	
	1986	
	1982	
	1981	
	1979	
EXPSPACE-hard (Lipton)	1976	

Known results timeline

VASS		2-VASS
	2015	
	2013	
	2012	
	2011	
	2009	
	2004	
	1992	
	1986	
	1982	
	1981	
	1979	Decidable (Hopcroft & Pansiot)
EXPSPACE-hard (Lipton)	1976	$\{\mathbf{v} : p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v})\}$ semilinear

Known results timeline

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		2015	
		2013	
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	2013	
	2012	
	2011	
	2009	
	2004	
	1992	
	1986	2-EXPTIME (Howell, Huynh, Rosier & Yen)
Decidable (Kosaraju)	1982	
Decidable (Mayr)	1981	
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	2015	
	2013	
	2012	
	2011	
	2009	
	2004	
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	1986	2-EXPTIME (Howell, Huynh, Rosier & Yen)
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VASS		2-VASS
	2015	
	2013	
	2012	
	2011	
	2009	
	2004	$\{(\mathbf{u}, \mathbf{v}) : p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v})\}$ semilinear (Leroux & Sutre)
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	2012	
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Known results timeline

VASS		2-VASS
$\mathbf{F}_{\omega^3}$ (Leroux & Schmitz) in two talks!	2015	
	2013	PSPACE-hard (Fearnley & Jurdziński)
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Decidable (Leroux)	2011	
	2009	
	2004	$\{(\mathbf{u},\mathbf{v}) : p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v})\}$ semilinear (Leroux & Sutre)
Decidable (Lambert)	1992	
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Known results timeline

VASS	Year	2-VASS
$\mathbf{F}_{\omega^3}$ (Leroux & Schmitz) in two talks!	2015	**PSPACE-complete** (our contribution)
	2013	PSPACE-hard (Fearnley & Jurdziński)
	2012	
Decidable (Leroux)	2011	
	2009	
	2004	$\{(\mathbf{u},\mathbf{v}) : p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v})\}$ semilinear (Leroux & Sutre)
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Our main theorem

There exists $c \in \mathbb{N}$ s.t. for every 2-VASS V

$$p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v}) \implies p(\mathbf{u}) \xrightarrow{\pi} q(\mathbf{v}) \quad \text{where} \quad |\pi| \leq c^{|V|}$$

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Corollary

Reachability for 2-VASS $\in$ PSPACE

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Corollary: proof

Exp. length runs $\implies$ exp. intermediate counter values

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Corollary: proof

Exp. length runs $\implies$ exp. intermediate counter values

$\implies$ poly. size intermediate counter values

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Corollary: proof

Exp. length runs $\implies$ exp. intermediate counter values

$\implies$ poly. size intermediate counter values

$\implies$ guess run on the fly

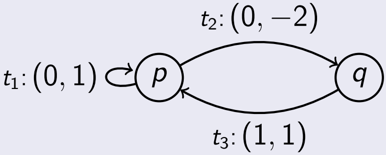
Our main theorem

There exists $c \in \mathbb{N}$ s.t. for every 2-VASS V

$$p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v}) \implies p(\mathbf{u}) \xrightarrow{\pi} q(\mathbf{v}) \quad \text{where} \quad |\pi| \leq c^{|V|}$$

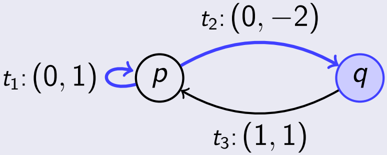
How to prove this theorem?

Bounding the runs



Runs from p to q:

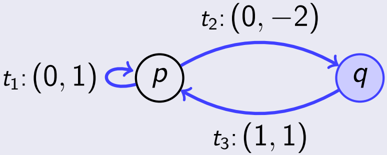
Bounding the runs



Runs from p to q:

$t_1{}^* t_2$

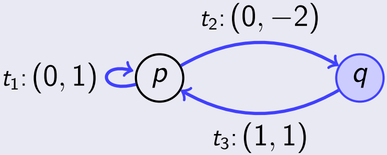
Bounding the runs



Runs from p to q:

$$t_1{}^* t_2 \ (t_3 t_1{}^* t_2)$$

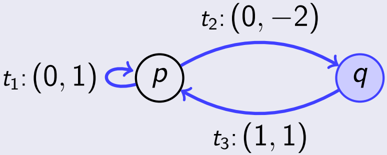
Bounding the runs



Runs from p to q:

$$t_1{}^* t_2 \; (t_3 t_1{}^* t_2) \; (t_3 t_1{}^* t_2)$$

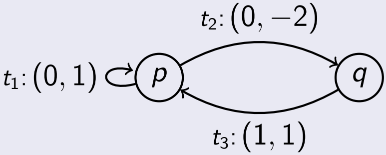
Bounding the runs



Runs from p to q:

$$t_1{}^* t_2 \; (t_3 t_1{}^* t_2) \; (t_3 t_1{}^* t_2) \cdots (t_3 t_1{}^* t_2)$$

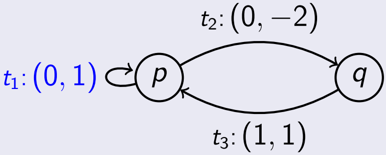
Bounding the runs



Runs from p to q:

$$t_1{}^* t_2 \ (t_3 t_1{}^* t_2) \ (t_3 t_1{}^* t_2) \cdots \ (t_3 t_1{}^* t_2)$$

Bounding the runs



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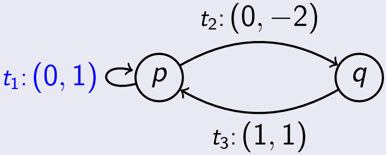
$$t_1{}^* t_2 \ (t_3 t_1{}^* t_2) \ (t_3 t_1{}^* t_2) \cdots \ (t_3 t_1{}^* t_2)$$

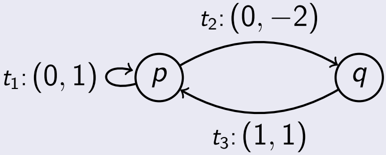
Bounding the runs



Runs from p to q:

$$\boldsymbol{t_1}^* t_2 \ (t_3 \quad t_2) \ (t_3 \quad t_2) \cdots (t_3 \quad t_2)$$

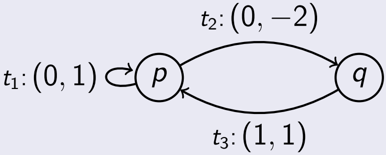
Bounding the runs



Runs from p to q:

$$t_1{}^* t_2 \; (t_3 \quad t_2) \; (t_3 \quad t_2) \cdots (t_3 \quad t_2)$$

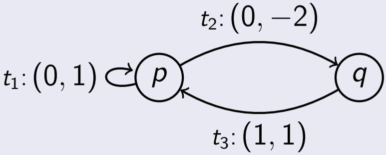
Bounding the runs



Runs from p to q:

$t_1{}^* t_2 \; (t_3 \quad t_2)^*$

Bounding the runs



Runs from p to q:

$$t_1{}^* t_2 \ (t_3 \quad t_2)^*$$

2-VASS can always be flattened (Leroux & Sutre '04)

$$\exists S = \bigcup_{\text{finite}} \alpha_0 {\beta_1}^* \alpha_1 \cdots {\beta_k}^* \alpha_k$$

2-VASS can always be flattened (Leroux & Sutre '04)

$$\exists S = \bigcup_{\text{finite}} \underbrace{\alpha_0 {\beta_1}^* \alpha_1 \cdots {\beta_k}^* \alpha_k}_{\textit{linear path scheme}}$$

2-VASS can always be flattened (Leroux & Sutre '04)

$\exists S = \bigcup_{\text{finite}} \underbrace{\alpha_0 {\beta_1}^* \alpha_1 \cdots {\beta_k}^* \alpha_k}_{\textit{linear path scheme}}$ such that

$$p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v}) \implies p(\mathbf{u}) \xrightarrow{\pi \in S} q(\mathbf{v})$$

2-VASS can always be flattened (Leroux & Sutre '04)

$$\exists S = \bigcup_{\text{finite}} \underbrace{\alpha_0 {\beta_1}^* \alpha_1 \cdots {\beta_k}^* \alpha_k}_{\textit{linear path scheme}} \text{ such that}$$

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2-VASS have small linear path schemes (our contribution)

- $|\alpha_i|, |\beta_i| \quad \leq (|Q| + \|T\|)^{O(1)}$

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2-VASS have small linear path schemes (our contribution)

- $|\alpha_i|, |\beta_i| \quad \leq (|Q| + \|T\|)^{O(1)}$
- $k \quad \in O(|Q|^2)$

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2-VASS have small linear path schemes (our contribution)

- $|\alpha_i|, |\beta_i| \leq (|Q| + \|T\|)^{O(1)}$
- $k \in O(|Q|^2)$
- $*$-exponents $\leq (|Q| + \|T\| + \|\mathbf{u}\| + \|\mathbf{v}\|)^{O(1)}$

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$\exists S = \bigcup_{\text{finite}} \underbrace{\alpha_0 {\beta_1}^* \alpha_1 \cdots {\beta_k}^* \alpha_k}_{\textit{linear path scheme}}$ such that

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2-VASS have small linear path schemes (our contribution)

- $|\alpha_i|, |\beta_i| \leq$ exponential
- $k \in$ polynomial
- $*$-exponents $\leq$ exponential

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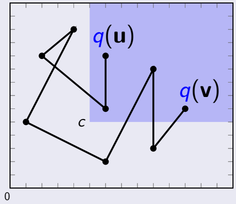
$$\exists S = \bigcup_{\text{finite}} \underbrace{\alpha_0 {\beta_1}^* \alpha_1 \cdots {\beta_k}^* \alpha_k}_{\textit{linear path scheme}} \text{ such that}$$

$$p(\mathbf{u}) \xrightarrow{*} q(\mathbf{v}) \implies p(\mathbf{u}) \xrightarrow{\pi \in S} q(\mathbf{v})$$

How to obtain such linear path schemes?

Obtaining linear path schemes for 3 types of runs

Type 1



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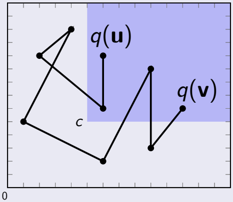

Remove zigzags

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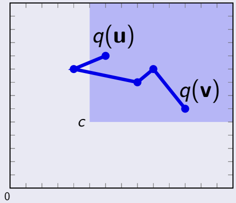
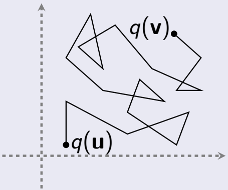


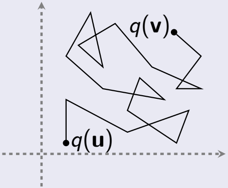
Remove zigzags

Type 1: removing zig-zags

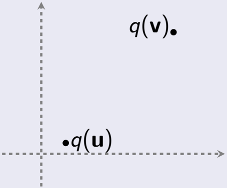


Arbitrary run

Type 1: removing zig-zags



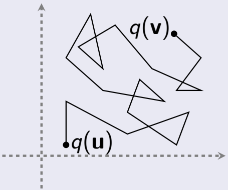
$\longrightarrow$



Arbitrary run

$$\bigcup_{\text{finite}} \alpha_0 \beta_1^* \alpha_1 \cdots \beta_k^* \alpha_k$$

Type 1: removing zig-zags



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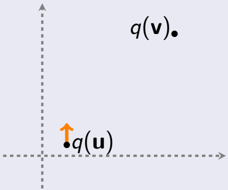
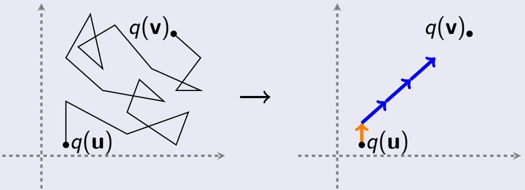


Arbitrary run $\quad \bigcup_{\text{finite}} \alpha_0 \beta_1^* \alpha_1 \cdots \beta_k^* \alpha_k$

Type 1: removing zig-zags



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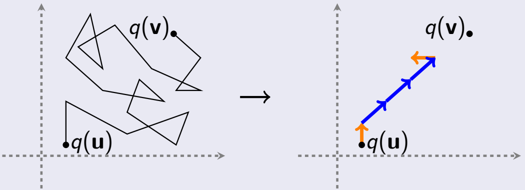

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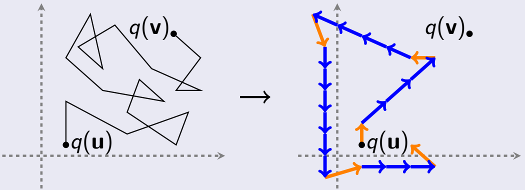

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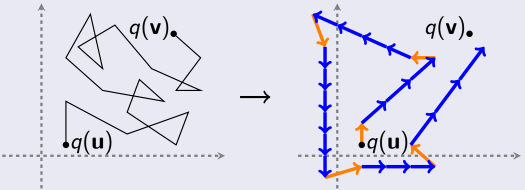

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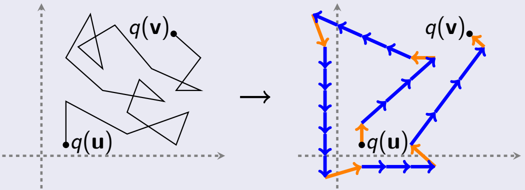

Arbitrary run $\qquad \bigcup_{\text{finite}} \alpha_0\beta_1^*\alpha_1\cdots\beta_k^*\alpha_k$

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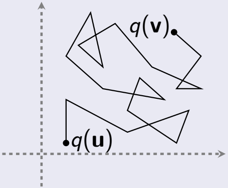

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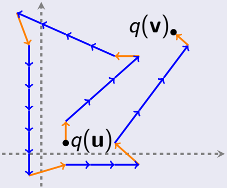

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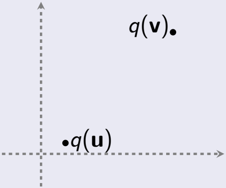

Arbitrary run

$\bigcup_{\text{finite}} \alpha_0 \beta_1^* \alpha_1 \cdots \beta_k^* \alpha_k$

$\bigcup_{\text{finite}} \gamma_0 \boldsymbol{\theta}_1^* \gamma_1 \boldsymbol{\theta}_2^* \gamma_2$

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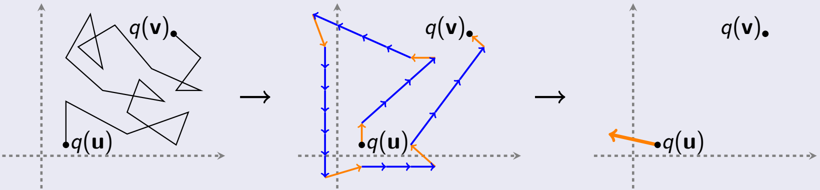

Arbitrary run $\quad \bigcup_{\text{finite}} \alpha_0\beta_1^*\alpha_1\cdots\beta_k^*\alpha_k \quad \bigcup_{\text{finite}} \gamma_0\boldsymbol{\theta}_1^*\gamma_1\boldsymbol{\theta}_2^*\gamma_2$

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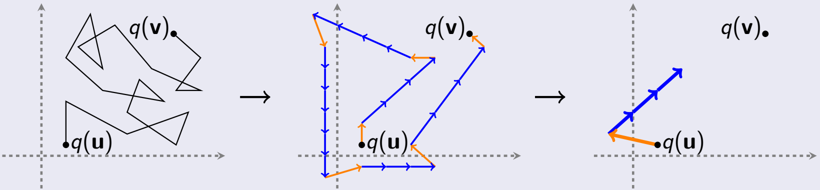

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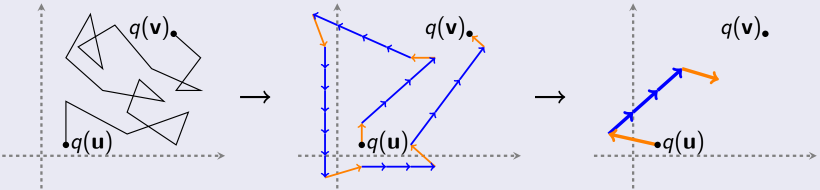

Arbitrary run

$$\bigcup_{\text{finite}} \alpha_0\beta_1^*\alpha_1\cdots\beta_k^*\alpha_k$$

$$\bigcup_{\text{finite}} \gamma_0\boldsymbol{\theta}_1^*\gamma_1\boldsymbol{\theta}_2^*\gamma_2$$

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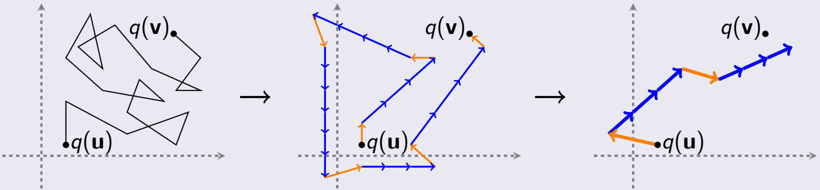
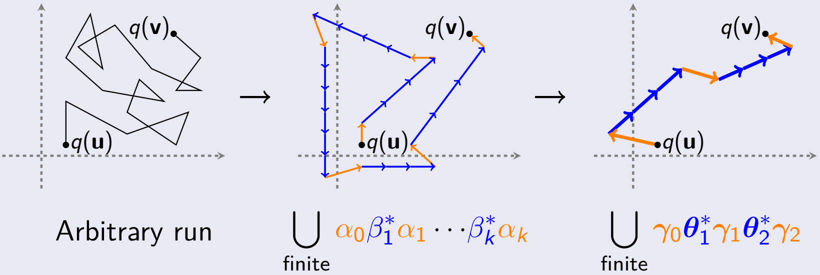
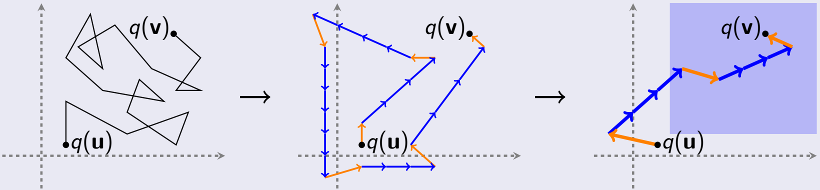


Arbitrary run $\longrightarrow$ $\bigcup_{\text{finite}} \alpha_0 \beta_1^* \alpha_1 \cdots \beta_k^* \alpha_k$ $\longrightarrow$ $\bigcup_{\text{finite}} \gamma_0 \boldsymbol{\theta}_1^* \gamma_1 \boldsymbol{\theta}_2^* \gamma_2$

Type 1: removing zig-zags



Type 1: removing zig-zags



Arbitrary run $\quad \bigcup_{\text{finite}} \alpha_0\beta_1^*\alpha_1\cdots\beta_k^*\alpha_k \quad \bigcup_{\text{finite}} \gamma_0\boldsymbol{\theta}_1^*\gamma_1\boldsymbol{\theta}_2^*\gamma_2$

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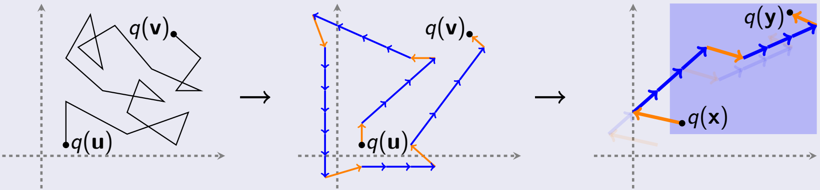

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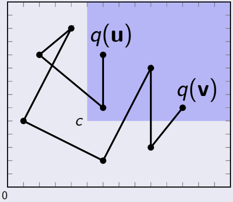
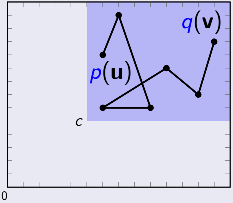


Type 2



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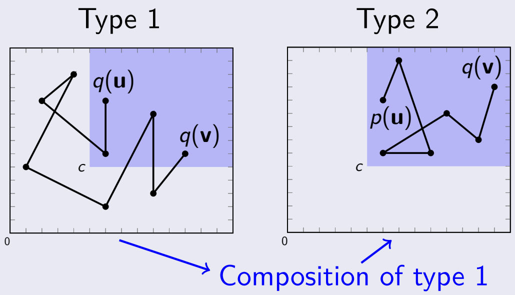
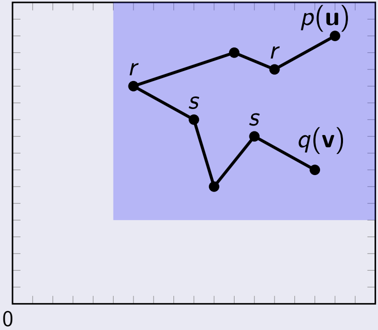


Type 2: decomposition



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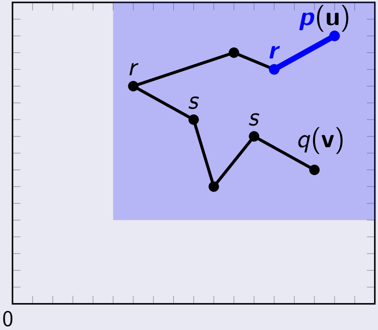
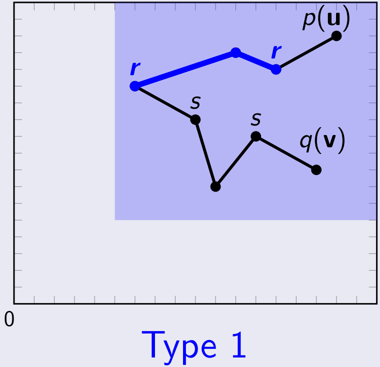


Small run

Type 2: decomposition



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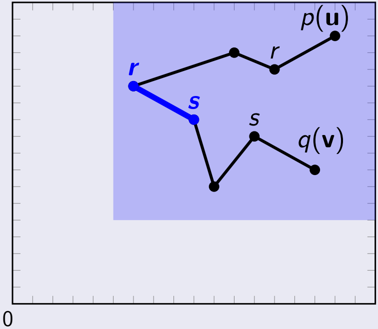

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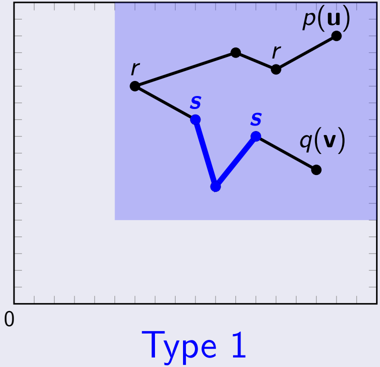

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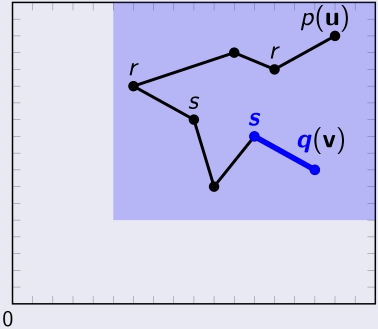
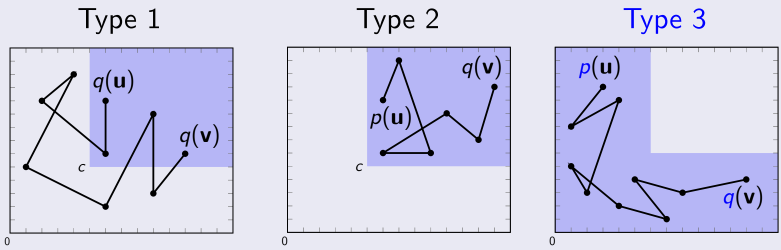


Small run

Obtaining linear path schemes for 3 types of runs



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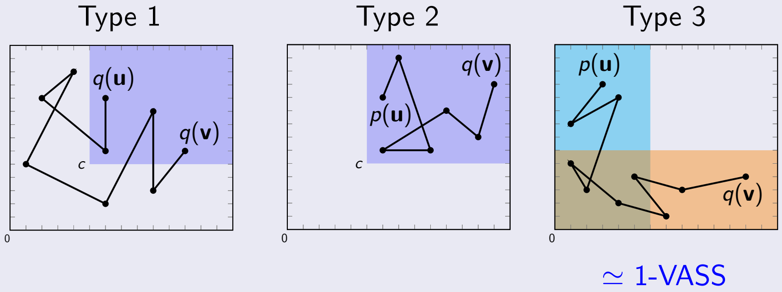
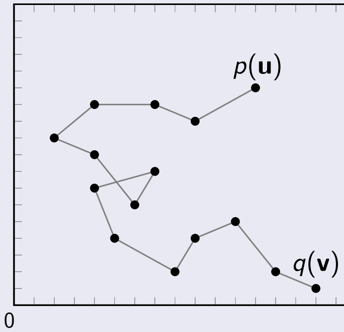


Every run decomposes into $\leq |Q| + 1$ runs of type 1, 2 & 3



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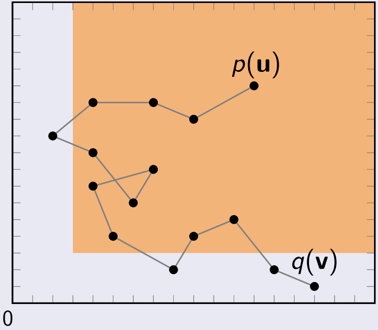

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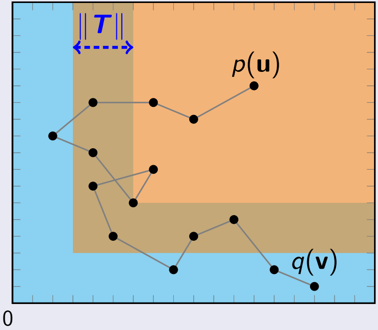

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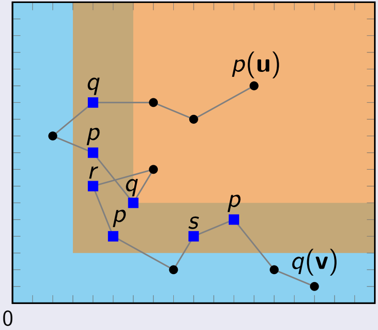

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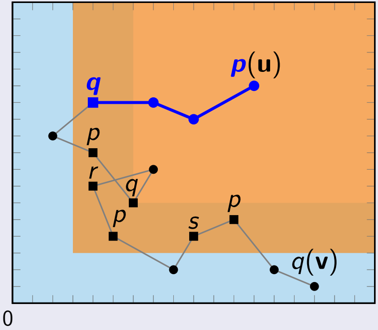
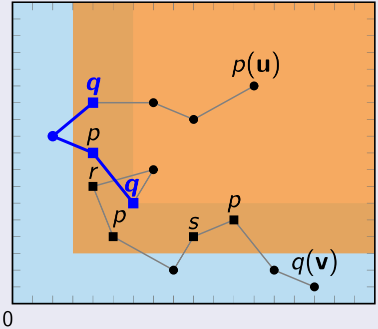


Type 2

Every run decomposes into $\leq |Q| + 1$ runs of type 1, 2 & 3



Type 1

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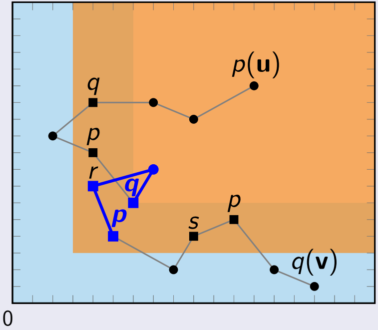

Type 2

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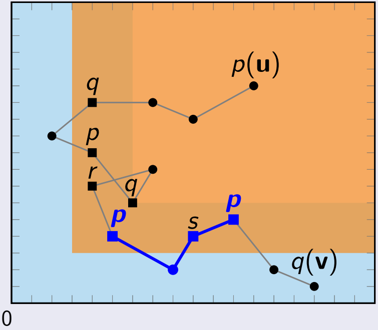

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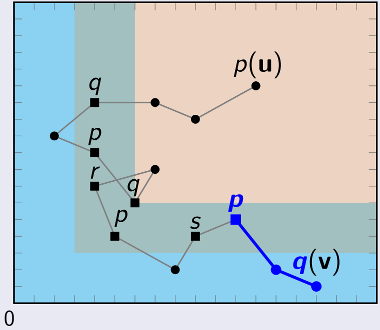

Type 3

Open questions

- 2-VASS, unary encoding: NL-hard and $\in$ NP. NL-complete?

Open questions

- 2-VASS, unary encoding: NL-hard and $\in$ NP. NL-complete?
- 3-VASS: PSPACE-hard and $\in \mathbf{F}_{\omega^3}$. Better bounds?

Thank you!

ありがとうございます!

(arigatō gozaimasu)