PhD Open
Algorithmic verification of infinite-state systems via relaxations

Homework

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Due on:	Dec. 1st, 2022 at 23:59 (Warsaw time)
Submit:	to michael.blondin@usherbrooke.ca
Grade:	over 50 points

Question 1.

Consider the Dekker mutual exclusion algorithm below, where two processes share variables

$$\texttt{enter[0]}, \texttt{enter[1]} \in \{\texttt{false}, \texttt{true}\} \text{ and } \texttt{turn} \in \{0, 1\}.$$

Variables `enter[0]` and `enter[1]` are initialized to `false`, and `turn` can be initialized to either 0 or 1. The algorithm is made of two similar processes:

Process 0

```
while true:
  enter[0] = true
  while enter[1]:
    if turn ≠ 0:
      enter[0] = false
      while turn ≠ 0: pass
      enter[0] = true
  /* critical section */
  turn = 1
  enter[0] = false
```

Process 1

```
while true:
  enter[1] = true
  while enter[0]:
    if turn ≠ 1:
      enter[1] = false
      while turn ≠ 1: pass
      enter[1] = true
  /* critical section */
  turn = 0
  enter[1] = false
```

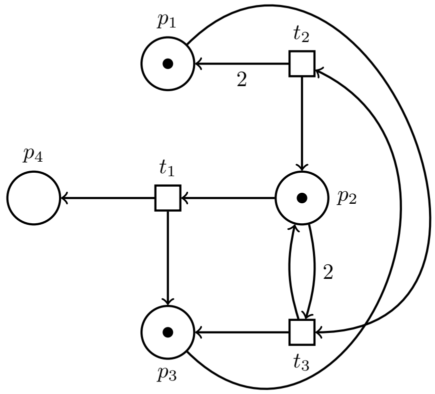
You must partially model the Dekker algorithm with a Petri net $\mathcal{N}$, as done in class:

(a) Draw the places of $\mathcal{N}$. *1 pt*

(b) Draw six transitions associated to process 0. *2.5 pts*

(c) Say what property of $\mathcal{N}$ must be verified in order to determine whether the two processes can both end up in their critical section (at the same time). *2.5 pts*

Question 2.
Let $\mathcal{N} = (P, T, F)$ be the following Petri net:



(a) Construct a coverability graph of $\mathcal{N}$ from marking $\boldsymbol{m}_{\text{src}} := (1, 1, 1, 0)$. 2.5 pts

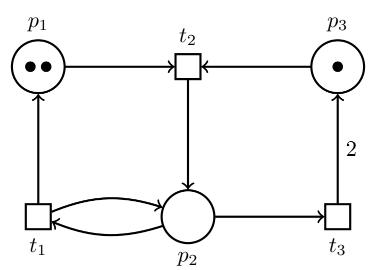
(b) Say whether $\text{Post}^*(\boldsymbol{m}_{\text{src}})$ is infinite or not. Justify. 2 pts

(c) Say, with a brief justification, which of these markings are coverable from $\boldsymbol{m}_{\text{src}}$: 1.5 pts

- $\boldsymbol{m}_1 := (5, 0, 2, 3)$,
- $\boldsymbol{m}_2 := (99, 2, 1, 0)$,
- $\boldsymbol{m}_3 := (3, 2, 0, 1)$.

(d) Say whether it is possible to fire t_3 infinitely often from $\boldsymbol{m}_{\text{src}}$. Justify. 2 pts

Question 3.
Let $\mathcal{N} = (P, T, F)$ be the following Petri net:



(a) Compute a minimal basis of $\uparrow\text{Pre}^*(\uparrow(2, 0, 1))$, i.e. the set of markings that can cover $(2, 0, 1)$. Leave a trace of your computations. 4.5 pts

(b) Determine, with a brief justification, which of these markings can cover $(2, 0, 1)$: 1.5 pts

- $\boldsymbol{m}_1 := (1, 1, 0)$,
- $\boldsymbol{m}_2 := (3, 2, 6)$,
- $\boldsymbol{m}_3 := (1, 99, 42)$.

Question 4.

A *Petri net with capacities* $\mathcal{N} = (P, T, F, \boldsymbol{c})$ is a Petri net (P, T, F) where each place p has a capacity $\boldsymbol{c}(p) \in \mathbb{N} \cup \{\infty\}$ that indicates the maximal number of tokens that place p may hold. In this formalism, a transition t is firable from a marking $\boldsymbol{m}$ iff 6 pts

$$\boldsymbol{m}(p) \geq F(p,t) \text{ and } \boldsymbol{m}(p) - F(p,t) + F(t,p) \leq \boldsymbol{c}(p).$$

A standard Petri net corresponds to the case where $\boldsymbol{c} = (\infty, \ldots, \infty)$.

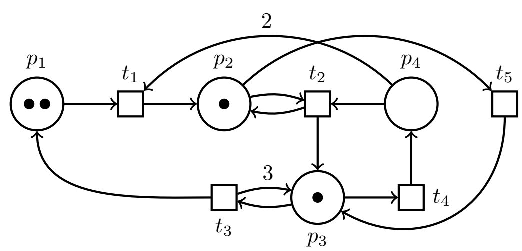
Show that the reachability problem for Petri nets with capacities can be solved through the reachability problem for standard Petri nets. More precisely, explain how to convert, in polynomial time, an input $(\mathcal{N}, \boldsymbol{m}, \boldsymbol{m}')$, where $\mathcal{N}$ has capacities, into an input $(\mathcal{N}', \boldsymbol{x}, \boldsymbol{x}')$, where $\mathcal{N}'$ has no capacities, and such that

$$\boldsymbol{m} \xrightarrow{*} \boldsymbol{m}' \text{ in } \mathcal{N} \iff \boldsymbol{x} \xrightarrow{*} \boldsymbol{x}' \text{ in } \mathcal{N}'.$$

(In the terminology of computational complexity theory, you must give a polynomial-time many-one reduction, from the reachability problem for Petri nets with capacities, to the reachability problem for standard Petri nets.)

Question 5.

Consider the following Petri net:



Let $\boldsymbol{m}_{\text{src}} := (2, 1, 1, 0)$ and $\boldsymbol{m}_{\text{tgt}} := (2, 0, 0, 1)$. Say, with justification, whether these statements hold: 10 pts

(a) $\boldsymbol{m}_{\text{src}} \overset{*}{\dashrightarrow\!\!\!\!\twoheadrightarrow} \boldsymbol{m}_{\text{tgt}}$,

(b) $\boldsymbol{m}_{\text{src}} \overset{*}{\dashrightarrow} \boldsymbol{m}_{\text{tgt}}$,

(c) $\boldsymbol{m}_{\text{src}} \overset{*}{\twoheadrightarrow} \boldsymbol{m}_{\text{tgt}}$,

(d) $\boldsymbol{m}_{\text{src}} \xrightarrow{*} \boldsymbol{m}_{\text{tgt}}$.

Question 6.

Show that the two following problems are decidable, i.e. that they can both be solved with algorithms: 6 pts

<u>CONTINUOUS COVERABILITY PROBLEM</u>

INPUT: Petri net $\mathcal{N} = (P, T, F)$ and markings $\boldsymbol{m}_{\text{src}}, \boldsymbol{m}_{\text{tgt}} \in \mathbb{N}^P$

QUESTION: $\exists \boldsymbol{m} \in \mathbb{R}^P_{\geq 0} : \boldsymbol{m}_{\text{src}} \overset{*}{\twoheadrightarrow} \boldsymbol{m}$ and $\boldsymbol{m} \geq \boldsymbol{m}_{\text{tgt}}$?

<u>CONTINUOUS INCLUSION PROBLEM</u>

INPUT: Petri nets $\mathcal{N}$ and $\mathcal{N}'$ over the same set of places P, and markings $\boldsymbol{m}_{\text{src}}, \boldsymbol{m}'_{\text{src}} \in \mathbb{N}^P$

QUESTION: $\{\boldsymbol{m} \in \mathbb{R}^P_{\geq 0} : \boldsymbol{m}_{\text{src}} \overset{*}{\twoheadrightarrow} \boldsymbol{m} \text{ in } \mathcal{N}\} \subseteq \{\boldsymbol{m}' \in \mathbb{R}^P_{\geq 0} : \boldsymbol{m}'_{\text{src}} \overset{*}{\twoheadrightarrow} \boldsymbol{m}' \text{ in } \mathcal{N}'\}$?

(Hint: logic.)

Question 7.
Recall the marking equation defined as

$$\exists \boldsymbol{x} \in \mathbb{N}^T : \boldsymbol{m}_{\text{src}} + \sum_{t \in T} \boldsymbol{\Delta}(t) \cdot \boldsymbol{x}(t) = \boldsymbol{m}_{\text{tgt}},$$

where $\boldsymbol{\Delta}(t) \in \mathbb{N}^P$ is such that $\boldsymbol{\Delta}(t)(p) \coloneqq F(t,p) - F(p,t)$ for every $p \in P$.

Let the *weak marking equation* be defined in exactly the same way, except that $\boldsymbol{x} \in \mathbb{R}^T$, i.e. where entries of $\boldsymbol{x}$ can be (possibly negative) real numbers. Let $d \colon \mathbb{N}^P \times \mathbb{N}^P \to \mathbb{R} \cup \{-\infty, \infty\}$ be such that

$$d(\boldsymbol{m}, \boldsymbol{m}') \coloneqq \min \left\{ \sum_{t \in T} \boldsymbol{x}(t) : \boldsymbol{x} \text{ is a solution to the weak marking equation with } \boldsymbol{m}_{\text{src}} = \boldsymbol{m} \text{ and } \boldsymbol{m}_{\text{tgt}} = \boldsymbol{m}' \right\}.$$

Consider the heuristic $h(\boldsymbol{m}) \coloneqq \max(d(\boldsymbol{m}, \boldsymbol{m}_{\text{tgt}}), 0)$. Say, with justifications, whether heuristic h is always:

(a) consistent, 4 pts

(b) unbounded. 4 pts