

		Cartesian product	$S \times T$		
		Power set	$\mathbb{P}(S)$		L
					L
		Comprehension 2	$\{x \mid P\}$		L
5	where $S$ and $T$ a	are sets, $x$ is a variable	and <i>P</i> is a predi	cate.	or Machichais Zürich
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Power Set	



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#### Cartesian Product



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Set Comprehension	



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#### Basic Set Operator Memberships (Axioms)

#### These axioms are defined by equivalences. Left Part Right Part $E \mapsto F \in S \times T$ $E \in S \land F \in T$ $S \in \mathbb{P}(T)$ $\forall x \cdot (x \in S \Rightarrow x \in T)$ (x is not free in S and T) $E \in \{x \mid P\}$ [x := E]P(x is not free in E) Hochschule Zürlei schnology Zurich ond Abrial (ETH-Zürich) Su nary of the Mathematical Notation Bucharest, 14-16/07/10 57 / 120

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ementary Set Operators	

		 e Hachschule Zürich
Empty set	Ø	
Extension	$\{a,\ldots,b\}$	
Difference	$S \setminus T$	
Intersection	$S \cap T$	
Union	$S \cup T$	

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# Set Inclusion and Extensionality Axiom

Left Part	Right Part
$S \subseteq T$	$S\in \mathbb{P}(\mathcal{T})$
S = T	$S \subseteq T \land T \subseteq S$

The first rule is just a syntactic extension

The second rule is the Extensionality Axiom



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# Elementary Set Operator Memberships

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*	E∈Ø	1		
	${\it E} \in \{{\it a},\ldots,{\it b}\}$	$E = a \lor \ldots \lor$	E = b	
	$E \in S \setminus T$	$E \in S \land E \notin T$		
	$E \in S \cap T$	$E \in S \land E \in T$		
	$E \in S \cup T$	$E \in S \lor E \in T$		

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Generalizations of Elementary	Operators

# Generalized Unionunion (S)Union Quantifier $\bigcup x \cdot (P \mid T)$ Generalized Intersectioninter (S)Intersection Quantifier $\bigcap x \cdot (P \mid T)$

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# Summary of Basic and Elementary Operators



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#### Generalized Intersection



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Summary of Generalizations c	f Elementary Operators

union (S)	
$\bigcup x \cdot P \mid T$	
inter ( $S$ )	
$\bigcap x \cdot P \mid T$	



		$E \in unic$	$\operatorname{on}(S)$	$\exists s \cdot s \in S \land E$ (s is not free in	$f \in s$ S and E)		
		E ∈ (∪	$x \cdot P \mid T$ )	$\exists x \cdot P \land E \in \\ (x \text{ is not free in})$	T n E)		
		$E \in \text{inter}$	r ( <i>S</i> )	$\forall s \cdot s \in S \Rightarrow a$ (s is not free in	$E \in s$ S and E)		
		E ∈ (∩	$x \cdot P \mid T$ )	$\forall x \cdot P \Rightarrow E \in$ (x is not free in	T n E)		
Tione .	Well-define Well-define	edness condi edness condi	tion for case tion for case	3: $S \neq \emptyset$ 4: $\exists x \cdot P$			J
<b>/</b>	Raymond Abrial	(ETH-Zürich)	Summary of the	Mathematical Notation	Bucharest, 1	Bidgendesheche Te Swiss Texteral Inst 4–16/07/10	schnische Hochschule Zürich litzte of Technology Zurich 66 / 120

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 A Quick Review of First Order Predicate Calculus

 A Refresher on Set Theory

 Binary Relation Operators (1)

Binary relations	$S \leftrightarrow T$	
Domain	dom ( <i>r</i> )	
Range	ran(r)	
Converse	r <sup>-1</sup>	





# A Binary Relation *r* from a Set A to a Set B



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Range of Binary Relation <i>r</i>		



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#### Domain of Binary Relation r



Basic Constructs Extensions A Quick Review A Quick Review of First Order Predicate Calculus A Refresher on Set Theory Converse of Binary Relation r



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Iculus Basic Constructs Iculus Extensions

# Binary Relation Operators (3)

Domain restriction	<i>S</i> ⊲ <i>r</i>	
Range restriction	<i>r</i> ⊳ <i>T</i>	
Domain subtraction	S⊲r	
Range subtraction	r ⊳ T	









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The Range Substraction Oper	rator	



#### The Domain Substraction Operator



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Left Part	Right Part	
$E\mapsto F \in S \lhd r$	$E \in S \land E \mapsto F \in r$	
$E\mapsto F\in r\rhd T$	$E\mapsto F\in r \land F\in T$	
$E\mapsto F\in S \lhd r$	$E \notin S \land E \mapsto F \in r$	
$E\mapsto F\in r \triangleright T$	$E\mapsto F\in r \land F\notin T$	

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# Binary Relation Operators (4)

Image	<i>r</i> [ <i>w</i> ]	
Composition	p;q	
Overriding	$p \Leftrightarrow q$	
Identity	id ( <i>S</i> )	

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Forward Composition	

# <image><image><image><image><image><image><image><image><image><image><image><image><image>

#### Foundation for Deductive and Formal Proofs A Quick Review of Propositional Calculus A Quick Review of Propositional Calculus A Refresher on Set Theory Image of {a5, a7} under r









# The Overriding Operator



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Special Case	



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# Special Case



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Iore classical Results	

Given a relation $r$ such that $r \in \mathcal{S} \leftrightarrow \mathcal{S}$	S
$r = r^{-1}$	r is symmetric
$r \cap r^{-1} = \emptyset$	r is asymmetric
$r \cap r^{-1} \subseteq \operatorname{id}(S)$	r is antisymmetric
$\operatorname{id}(S) \subseteq r$	r is reflexive
$r \cap \mathrm{id}(S) = \emptyset$	r is irreflexive
<i>r</i> ; <i>r</i> ⊆ <i>r</i>	r is transitive
	ETH: Internet in the second

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Set-theoretic statements are far more readable than predicate calculus statements



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Function Operators (1)	

Partial functions	$S \leftrightarrow T$	
Total functions	$S \to T$	
Partial injections	$S \rightarrowtail T$	
Total injections	$S \rightarrowtail T$	



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#### A Partial Function F from a Set A to a Set B



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A Partial Injection F from a S	Set A to a Set B



#### A Total Function F from a Set A to a Set B



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Basic Constructs Extensions A Quick Review of First Order Predicate Calculus A Refresher on Set Theory

# Function Operator Memberships (1)

	Left Part	Right Part	
	$f \in S \leftrightarrow T$	$f \in S \leftrightarrow T \land (f^{-1}; f) = \operatorname{id}(\operatorname{ran}(f))$	
	$f \in S \to T$	$f \in S  woheadrightarrow T \wedge s = \operatorname{dom}(f)$	
	$f \in S \rightarrowtail T$	$f \in S \Rightarrow T \land f^{-1} \in T \Rightarrow S$	
×	$f \in S \rightarrowtail T$	$f \in S \rightarrow T \land f^{-1} \in T \Rightarrow S$	
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Partial Surjection F from a	Set A to a Set B



#### Basic Constructs Extensions A Quick Review of First Order Predicate Calculus A Refresher on Set Theory

# Function Operators (2)

Partial surjections	S -+-> T	
Total surjections	S» T	
Bijections	$S \rightarrowtail T$	









# A Bijection F from a Set A to a Set B



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Summary of Function Operate	ors	





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# Function Operator Memberships (2)

Left Part	Right Part
$f \in S \twoheadrightarrow T$	$f \in S  woheadrightarrow T \wedge T = \operatorname{ran}(f)$
$f \in S \twoheadrightarrow T$	$f \in S  ightarrow T \land T = \operatorname{ran}(f)$
$f \in S  ightarrow T$	$f \in S  ightarrow T \land f \in S \twoheadrightarrow T$



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	Operators (40)

			· · · · · ·	1	1
$S \times T$	$S \setminus T$	$r^{-1}$	r[w]	id ( <i>S</i> )	$\{x   x \in S \land P\}$
₽( <i>S</i> )		S ⊲ r S ⊲ r	p;q	$S \leftrightarrow T$ $S \rightarrow T$	
$S \subseteq T$		r ⊳ T r ⊳ T	<i>p</i>	$\begin{array}{c} S \rightarrowtail T \\ S \rightarrowtail T \end{array}$	{ a, b,, n }
$S \cup T$	dom $(r)$ ran $(r)$			S ↔ T S → T	union U
$S \cap T$	Ø	-		$S \rightarrowtail T$	inter
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